Problem Set 4
Optical Waveguides and Fibers (OWF)

will be discussed and collected in the tutorial on December 02, 2015

IMPORTANT INFORMATION - PLEASE READ THIS PAGE CAREFULLY

This problem set is partially done individually and partially in team work. In order to assign you to a team, please get in contact with the tutors using the contact details below. Please note that this problem set will be collected and graded as a part of the bonus system. You can submit it until Wednesday, December 02, 2015 at 11:30 AM. On Wednesday, November 18, 2015 there will be a question and answer session about this problem set instead of the regular tutorial.

If you have any doubts/questions concerning anything about this problem set, please do not hesitate to get in contact with the tutors using the contact details below.

Questions and Comments:

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Team 1: TE-modes

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Exercise 1: Numerical solution for the asymmetric slab waveguide.

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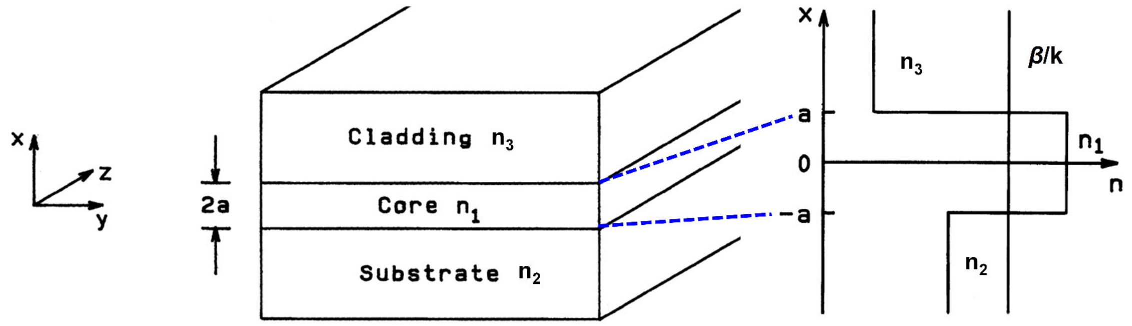

Figure 1: Slab waveguide with the definition of the coordinate system and the quantity a.

Guided modes of the slab waveguide assume the form

$$\underline{\mathbf{E}}(\mathbf{r},t) = \underline{\mathcal{E}}(x)e^{j(\omega t-\beta z)} \quad (1)$$

$$\underline{\mathbf{H}}(\mathbf{r},t) = \underline{\mathcal{H}}(x)e^{j(\omega t-\beta z)} \quad (2)$$

and can be classified in two categories: transverse electric (TE) modes, for which the electric field is purely transverse, i.e., only the components $\underline{\mathcal{E}}_y$, $\underline{\mathcal{H}}_x$ and $\underline{\mathcal{H}}_z$ are nonzero, and transverse magnetic (TM) modes, for which the magnetic field is purely transverse, i.e., the nonvanishing components are $\underline{\mathcal{H}}_y$, $\underline{\mathcal{E}}_x$ and $\underline{\mathcal{E}}_z$.

For the TE modes, the mode field $\underline{\mathcal{E}}_y(x)$ is given by

$$\underline{\mathcal{E}}_y(x) = \begin{cases} A\cos(k_{1x}a-\varphi)\exp\left(-k_{3x}^{(i)}(x-a)\right) & \text{for } x > a \\ A\cos(k_{1x}x-\varphi) & \text{for } -a \leq x \leq a \\ A\cos(-k_{1x}a-\varphi)\exp\left(k_{2x}^{(i)}(x+a)\right) & \text{for } x < -a \end{cases} \quad (3)$$

where

$$k_{1x} = \sqrt{n_1^2k_0^2-\beta^2}; \qquad k_{2x}^{(i)} = \sqrt{\beta^2-n_2^2k_0^2}; \qquad k_{3x}^{(i)} = \sqrt{\beta^2-n_3^2k_0^2}.$$

a) Starting from Eq. (3) calculate the corresponding $\underline{\mathcal{H}}_x$ and $\underline{\mathcal{H}}_z$ components.

b) From the continuity of $\underline{\mathcal{H}}_z$ at $x = \pm a$, derive an implicit equation for the mode's propagation constant β. As a result, you should obtain the relation

$$u \;= \tfrac{1}{2}\arctan\left(\tfrac{w}{u}\right) + \tfrac{1}{2}\arctan\left(\tfrac{w'}{u}\right) + \tfrac{m\pi}{2}, \tag{4}$$

where m is a non-negative integer, and where the parameters u, w, and w' are given by

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c) Write a progam, e.g., using MATLAB, which numerically solves Eq. (4) for arbitrary waveguide parameters and plots the electromagentic field components $\underline{\mathcal{E}}_y(x)$, $\underline{\mathcal{H}}_x(x)$, $\underline{\mathcal{H}}_z(x)$ for given mode number and frequency. To do so, insert a new set of parameters into Eq. (4),

$$u = V\sqrt{1-B}; \qquad w = V\sqrt{B}; \qquad w' = V\sqrt{\gamma + B}$$

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$$V = ak_0\sqrt{n_1^2 - n_2^2}; \qquad \gamma = \frac{n_2^2 - n_3^2}{n_1^2 - n_2^2}; \qquad B = \frac{\beta^2 - n_2^2k_0^2}{n_1^2k_0^2 - n_2^2k_0^2}.$$

By numerically solving the resulting equation, the normalized propagation constant B can be calculated for a given normalized frequency V and asymmetry parameter γ.

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d) Assume a slab waveguide in a silicon-on-insulator (SOI) structure, which consists of a silicon ($n_1 = 3.48$) layer on a silica substrate ($n_2 = 1.44$), covered by an air cladding ($n_3 = 1$). Assume that the refractive indices do not depend on frequency (no material dispersion). The silicon core is $2a = 500\,\text{nm}$ thick. For all the TE modes of the slab, plot the effective index $n_e = \beta/k_0$ and the normalized propagation constant B vs. frequency in the wavelength range between $1.12\,\mu\text{m}$ and $3.7\,\mu\text{m}$. In your plots, use both the real frequency f in THz and the normalized frequency V.

e) Assume now a symmetric doped-silica slab with $n_2 = n_3 = 1.45$ and $n_1 = n_2 + \delta n$ with $\delta n = 0.005$. The thickness is $2a = 4\,\mu\text{m}$. For the fundamental mode ($m = 0$), plot the effective refractive index n_e, the effective group refractive index n_eg, and the waveguide dispersion W_λ (unit: ps/(km nm)) in the range between $0.4\,\mu\text{m}$ and $3.7\,\mu\text{m}$. Repeat the plots for $a \to a/\sqrt{10}$ and $\delta n \to 10\delta n$ and discuss the differences.

f) Now take into account the material dispersion of silica. The cladding indices $n_2 = n_3$ can be obtained from the Sellmeier Eq. (5). This is the same equation that we used in problem set 2; it is valid between $0.2\,\mu\text{m}$ and $3.7\,\mu\text{m}$. For the core index $n_1 = n_2 + \delta n$, we assume that the index difference $\delta n = 0.005$ does not depend on frequency.

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Plot the chromatic dispersion C_λ (unit: ps/(km nm)) of the waveguide in the wavelength range between $0.4\,\mu\text{m}$ and $3.7\,\mu\text{m}$ and compare it to the results from part e). Discuss the differences.

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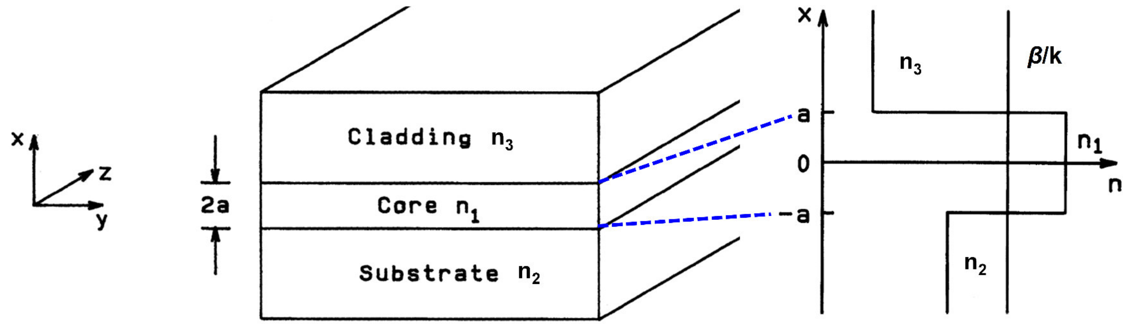

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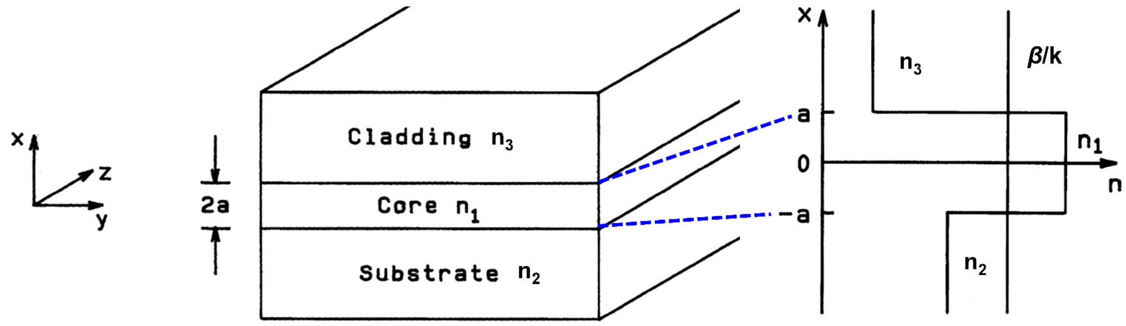

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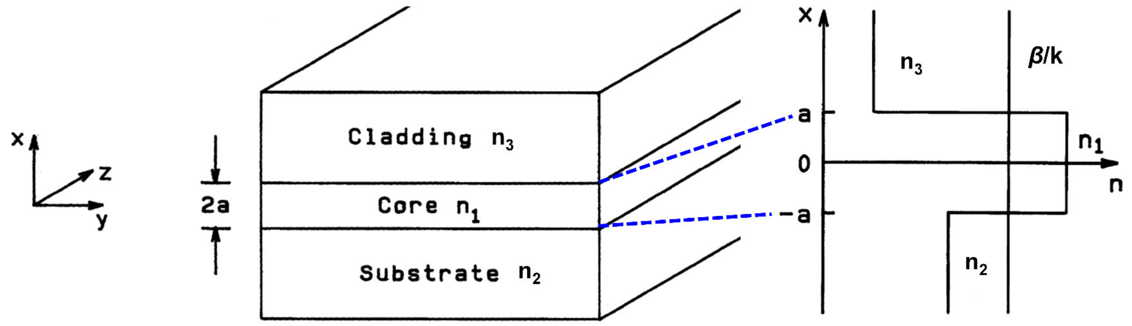

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d) Assume a slab waveguide in a silicon-on-insulator (SOI) structure, which consists of a silicon ($n_1 = 3.48$) layer on a silica substrate ($n_2 = 1.44$), covered by an air cladding ($n_3 = 1$). Assume that the refractive indices do not depend on frequency (no material dispersion). The silicon core is $2a = 400\,\text{nm}$ thick. For all the TE modes of the slab, plot the effective index $n_e = \beta/k_0$ and the normalized propagation constant B vs. frequency in the wavelength range between $1.12\,\mu\text{m}$ and $3.7\,\mu\text{m}$. In your plots, use both the real frequency f in THz and the normalized frequency V.

e) Assume now a symmetric doped-silica slab with $n_2 = n_3 = 1.45$ and $n_1 = n_2 + \delta n$ with $\delta n = 0.01$. The thickness is $2a = 4\,\mu\text{m}$. For the fundamental mode ($m = 0$), plot the effective refractive index n_e, the effective group refractive index n_eg, and the waveguide dispersion W_λ (unit: ps/(km nm)) in the range between $0.4\,\mu\text{m}$ and $3.7\,\mu\text{m}$. Repeat the plots for $a \to a/\sqrt{10}$ and $\delta n \to 10\delta n$ and discuss the differences.

f) Now take into account the material dispersion of silica. The cladding indices $n_2 = n_3$ can be obtained from the Sellmeier Eq. (5). This is the same equation that we used in problem set 2; it is valid between $0.2\,\mu\text{m}$ and $3.7\,\mu\text{m}$. For the core index $n_1 = n_2 + \delta n$, we assume that the index difference $\delta n = 0.005$ does not depend on frequency.

$$n_2^2(\lambda) = n_3^2(\lambda) = 1 + \frac{0.6962\lambda^2}{\lambda^2 - (0.06840\,\mu\text{m})^2} + \frac{0.4079\lambda^2}{\lambda^2 - (0.1162\,\mu\text{m})^2} + \frac{0.8975\lambda^2}{\lambda^2 - (9.8962\,\mu\text{m})^2}. \tag{5}$$

Plot the chromatic dispersion C_λ (unit: ps/(km nm)) of the waveguide in the wavelength range between $0.4\,\mu\text{m}$ and $3.7\,\mu\text{m}$ and compare it to the results from part e). Discuss the differences.

Questions and Comments:

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Problem Set 4
Optical Waveguides and Fibers (OWF)

will be discussed and collected in the tutorial on December 02, 2015

Team 5: TM-modes

This problem set aims at making a program (e.g., using MATLAB) that computes guided modes of asymmetric slab waveguides. Parts a) and b) shall be solved individually whereas parts c) to f) shall be solved in teams. This problem set will be collected and graded as a part of the bonus system. You can submit it until Wednesday, December 02, 2015, at 11:30 AM. On Wednesday, November 18, 2015, there will be a question and answer session about this problem set instead of the regular tutorial.

Exercise 1: Numerical solution for the asymmetric slab waveguide.

Consider a slab waveguide infinitely extended in the y- and z-direction as it is illustrated in Fig. 1, where n_1 is the refractive index of the core, n_2 and n_3 are the refractive indices of the substrate and the cladding, respectively, and where $n_3 \leq n_2$ holds without loss of generality.

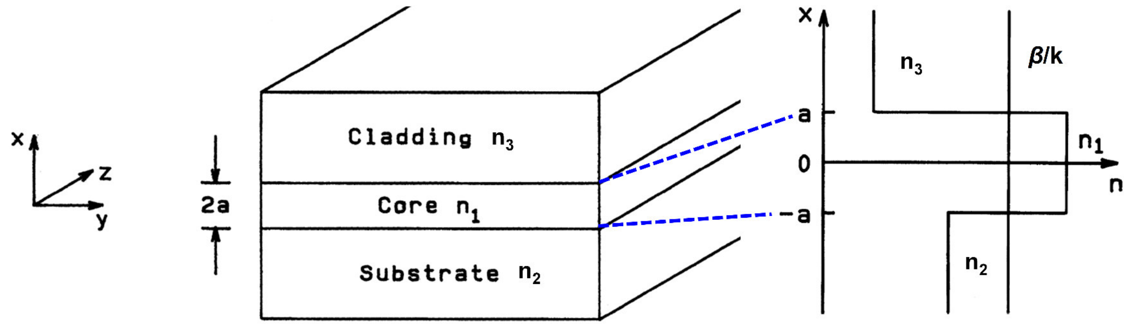

Figure 1: Slab waveguide with the definition of the coordinate system and the quantity a.

Guided modes of the slab waveguide assume the form

$$\underline{\mathbf{E}}(\mathbf{r},t) = \underline{\mathcal{E}}(x)e^{j(\omega t-\beta z)} \tag{1}$$

$$\underline{\mathbf{H}}(\mathbf{r},t) = \underline{\mathcal{H}}(x)e^{j(\omega t-\beta z)} \tag{2}$$

and can be classified in two categories: transverse electric (TE) modes, for which the electric field is purely transverse, i.e., only the components $\underline{\mathcal{E}}_y$, $\underline{\mathcal{H}}_x$ and $\underline{\mathcal{H}}_z$ are nonzero, and transverse magnetic (TM) modes, for which the magnetic field is purely transverse, i.e. the nonvanishing components are $\underline{\mathcal{H}}_y$, $\underline{\mathcal{E}}_x$ and $\underline{\mathcal{E}}_z$.

For the TM modes, the mode field $\underline{\mathcal{H}}_y(x)$ is given by

$$\underline{\mathcal{H}}_y(x) = \begin{cases} A\cos(k_{1x}a-\varphi)\exp\left(-k_{3x}^{(i)}(x-a)\right) & \text{for } x>a \\ A\cos(k_{1x}x-\varphi) & \text{for } -a\leq x\leq a \\ A\cos(-k_{1x}a-\varphi)\exp\left(k_{2x}^{(i)}(x+a)\right) & \text{for } x<-a \end{cases} \tag{3}$$

where

$$k_{1x}=\sqrt{n_1^2k_0^2-\beta^2}; \qquad k_{2x}^{(i)}=\sqrt{\beta^2-n_2^2k_0^2}; \qquad k_{3x}^{(i)}=\sqrt{\beta^2-n_3^2k_0^2}.$$

a) Starting from Eq. (3) calculate the corresponding $\underline{\mathcal{E}}_x$ and $\underline{\mathcal{E}}_z$ components.

b) From the continuity of $\underline{\mathcal{E}}_z$ at $x = \pm a$, derive an implicit equation for the mode's propagation constant β. As a result, you should obtain the relation

$$u = \frac{1}{2}\arctan\left(\frac{n_1^2 w}{n_2^2 u}\right) + \frac{1}{2}\arctan\left(\frac{n_1^2 w'}{n_3^2 u}\right) + \frac{m\pi}{2}, \quad (4)$$

where m is a non-negative integer, and where the parameters u, w, and w' are given by

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c) Write a progam, e.g., using MATLAB, which numerically solves Eq. (4) for arbitrary waveguide parameters and plots the electromagentic field components $\underline{\mathcal{H}}_y(x)$, $\underline{\mathcal{E}}_x(x)$, $\underline{\mathcal{E}}_z(x)$ for given mode number and frequency. To do so, insert a new set of parameters into Eq. (4),

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d) Assume a slab waveguide in a silicon-on-insulator (SOI) structure, which consists of a silicon ($n_1 = 3.48$) layer on a silica substrate ($n_2 = 1.44$), covered by an air cladding ($n_3 = 1$). Assume that the refractive indices do not depend on frequency (no material dispersion). The silicon core is $2a = 500\,\text{nm}$ thick. For all the TM modes of the slab, plot the effective index $n_e = \beta/k_0$ and the normalized propagation constant B vs. frequency in the wavelength range between $1.12\,\mu\text{m}$ and $3.7\,\mu\text{m}$. In your plots, use both the real frequency f in THz and the normalized frequency V.

e) Assume now a symmetric doped-silica slab with $n_2 = n_3 = 1.45$ and $n_1 = n_2 + \delta n$ with $\delta n = 0.005$. The thickness is $2a = 4\,\mu\text{m}$. For the fundamental mode ($m = 0$), plot the effective refractive index n_e, the effective group refractive index n_eg, and the waveguide dispersion W_λ (unit: ps/(km nm)) in the range between $0.4\,\mu\text{m}$ and $3.7\,\mu\text{m}$. Repeat the plots for $a \to a/\sqrt{10}$ and $\delta n \to 10\delta n$ and discuss the differences.

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Plot the chromatic dispersion C_λ (unit: ps/(km nm)) of the waveguide in the wavelength range between $0.4\,\mu\text{m}$ and $3.7\,\mu\text{m}$ and compare it to the results from part e). Discuss the differences.

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Problem Set 4
Optical Waveguides and Fibers (OWF)

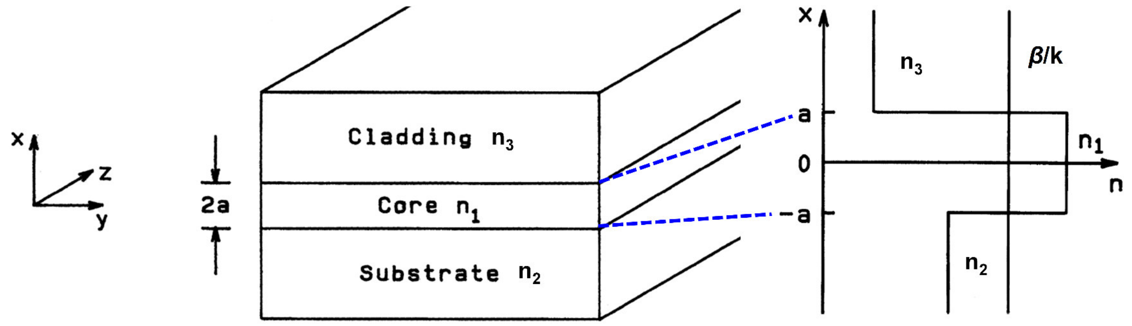
will be discussed and collected in the tutorial on December 02, 2015

Team 6: TM-modes

This problem set aims at making a program (e.g., using MATLAB) that computes guided modes of asymmetric slab waveguides. Parts a) and b) shall be solved individually whereas parts c) to f) shall be solved in teams. This problem set will be collected and graded as a part of the bonus system. You can submit it until Wednesday, December 02, 2015, at 11:30 AM. On Wednesday, November 18, 2015, there will be a question and answer session about this problem set instead of the regular tutorial.

Exercise 1: Numerical solution for the asymmetric slab waveguide.

Consider a slab waveguide infinitely extended in the y- and z-direction as it is illustrated in Fig. 1, where n_1 is the refractive index of the core, n_2 and n_3 are the refractive indices of the substrate and the cladding, respectively, and where $n_3 \leq n_2$ holds without loss of generality.



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where

$$k_{1x} = \sqrt{n_1^2 k_0^2 - \beta^2}; \qquad k_{2x}^{(i)} = \sqrt{\beta^2 - n_2^2 k_0^2}; \qquad k_{3x}^{(i)} = \sqrt{\beta^2 - n_3^2 k_0^2}.$$

a) Starting from Eq. (3) calculate the corresponding $\underline{\mathcal{E}}_x$ and $\underline{\mathcal{E}}_z$ components.

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c) Write a progam, e.g., using MATLAB, which numerically solves Eq. (4) for arbitrary waveguide parameters and plots the electromagentic field components $\underline{\mathcal{H}}_y(x)$, $\underline{\mathcal{E}}_x(x)$, $\underline{\mathcal{E}}_z(x)$ for given mode number and frequency. To do so, insert a new set of parameters into Eq. (4),

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Plot the chromatic dispersion C_λ (unit: ps/(km nm)) of the waveguide in the wavelength range between $0.4\,\mu\text{m}$ and $3.7\,\mu\text{m}$ and compare it to the results from part e). Discuss the differences.

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Problem Set 4
Optical Waveguides and Fibers (OWF)

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Team 7: TM-modes

This problem set aims at making a program (e.g., using MATLAB) that computes guided modes of asymmetric slab waveguides. Parts a) and b) shall be solved individually whereas parts c) to f) shall be solved in teams. This problem set will be collected and graded as a part of the bonus system. You can submit it until Wednesday, December 02, 2015, at 11:30 AM. On Wednesday, November 18, 2015, there will be a question and answer session about this problem set instead of the regular tutorial.

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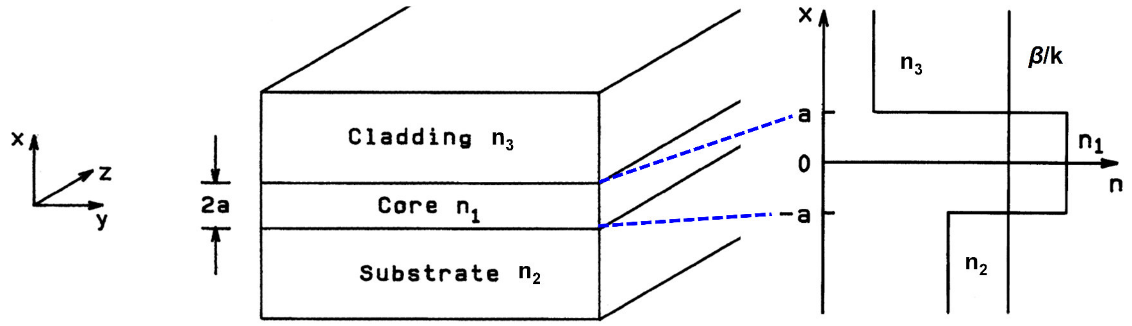

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Plot the chromatic dispersion C_λ (unit: ps/(km nm)) of the waveguide in the wavelength range between $0.4\,\mu\text{m}$ and $3.7\,\mu\text{m}$ and compare it to the results from part e). Discuss the differences.

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Optical Waveguides and Fibers (OWF)

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Team 8: TM-modes

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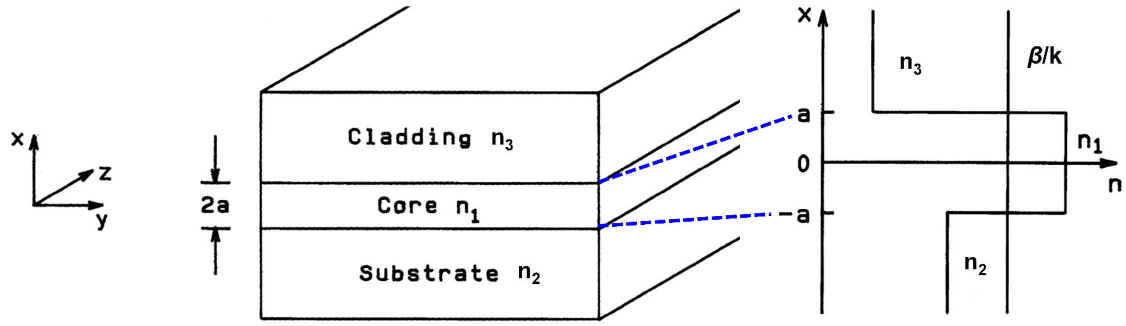

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$$k_{1x}=\sqrt{n_1^2k_0^2-\beta^2}; \qquad k_{2x}^{(i)}=\sqrt{\beta^2-n_2^2k_0^2}; \qquad k_{3x}^{(i)}=\sqrt{\beta^2-n_3^2k_0^2}.$$

a) Starting from Eq. (3) calculate the corresponding $\underline{\mathcal{E}}_x$ and $\underline{\mathcal{E}}_z$ components.

b) From the continuity of $\underline{\mathcal{E}}_z$ at $x = \pm a$, derive an implicit equation for the mode's propagation constant β. As a result, you should obtain the relation

$$u = \tfrac{1}{2}\arctan\left(\tfrac{n_1^2 w}{n_2^2 u}\right) + \tfrac{1}{2}\arctan\left(\tfrac{n_1^2 w'}{n_3^2 u}\right) + \tfrac{m\pi}{2}, \quad (4)$$

where m is a non-negative integer, and where the parameters u, w, and w' are given by

$$u = k_{1x}a = a\sqrt{n_1^2 k_0^2 - \beta^2}; \qquad w = k_{2x}^{(i)}a = a\sqrt{\beta^2 - n_2^2 k_0^2}; \qquad w' = k_{3x}^{(i)}a = a\sqrt{\beta^2 - n_3^2 k_0^2}.$$

c) Write a progam, e.g., using MATLAB, which numerically solves Eq. (4) for arbitrary waveguide parameters and plots the electromagentic field components $\underline{\mathcal{H}}_y(x)$, $\underline{\mathcal{E}}_x(x)$, $\underline{\mathcal{E}}_z(x)$ for given mode number and frequency. To do so, insert a new set of parameters into Eq. (4),

$$u = V\sqrt{1-B}; \qquad w = V\sqrt{B}; \qquad w' = V\sqrt{\gamma + B}$$

where

$$V = ak_0\sqrt{n_1^2 - n_2^2}; \qquad \gamma = \frac{n_2^2 - n_3^2}{n_1^2 - n_2^2}; \qquad B = \frac{\beta^2 - n_2^2 k_0^2}{n_1^2 k_0^2 - n_2^2 k_0^2}.$$

By numerically solving the resulting equation, the normalized propagation constant B can be calculated for a given normalized frequency V and asymmetry parameter γ.

Hint: MATLAB can be accessed from any computer at the SCC. For home use, a licence can be downloaded by any student via the SCC: http://www.scc.kit.edu/produkte/3841.php.

d) Assume a slab waveguide in a silicon-on-insulator (SOI) structure, which consists of a silicon ($n_1 = 3.48$) layer on a silica substrate ($n_2 = 1.44$), covered by an air cladding ($n_3 = 1$). Assume that the refractive indices do not depend on frequency (no material dispersion). The silicon core is $2a = 400\,\text{nm}$ thick. For all the TM modes of the slab, plot the effective index $n_e = \beta/k_0$ and the normalized propagation constant B vs. frequency in the wavelength range between $1.12\,\mu\text{m}$ and $3.7\,\mu\text{m}$. In your plots, use both the real frequency f in THz and the normalized frequency V.

e) Assume now a symmetric doped-silica slab with $n_2 = n_3 = 1.45$ and $n_1 = n_2 + \delta n$ with $\delta n = 0.01$. The thickness is $2a = 4\,\mu\text{m}$. For the fundamental mode ($m = 0$), plot the effective refractive index n_e, the effective group refractive index n_eg, and the waveguide dispersion W_λ (unit: ps/(km nm)) in the range between $0.4\,\mu\text{m}$ and $3.7\,\mu\text{m}$. Repeat the plots for $a \to a/\sqrt{10}$ and $\delta n \to 10\delta n$ and discuss the differences.

f) Now take into account the material dispersion of silica. The cladding indices $n_2 = n_3$ can be obtained from the Sellmeier Eq. (5). This is the same equation that we used in problem set 2; it is valid between $0.2\,\mu\text{m}$ and $3.7\,\mu\text{m}$. For the core index $n_1 = n_2 + \delta n$, we assume that the index difference $\delta n = 0.01$ does not depend on frequency.

$$n_2^2(\lambda) = n_3^2(\lambda) = 1 + \frac{0.6962\lambda^2}{\lambda^2 - (0.06840\,\mu\text{m})^2} + \frac{0.4079\lambda^2}{\lambda^2 - (0.1162\,\mu\text{m})^2} + \frac{0.8975\lambda^2}{\lambda^2 - (9.8962\,\mu\text{m})^2}. \quad (5)$$

Plot the chromatic dispersion C_λ (unit: ps/(km nm)) of the waveguide in the wavelength range between $0.4\,\mu\text{m}$ and $3.7\,\mu\text{m}$ and compare it to the results from part e). Discuss the differences.

Questions and Comments:

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